EFFECT OF TORSO GEOMETRY ON THE MAGNETOCARDIOGRAM

JOHN H. TRIPP, *Department of Physics, Case Western Reserve University, Cleveland, Ohio 44106*

ABSTRACT Calculations of the effect of torso geometry on the extracorporeal magnetic field produced by a simple cardiac source have been carried out. Contrary to the results at present in the literature, it is found that the field solution is stable under perturbations of geometry in the sense that small relative changes in geometry produce comparably small changes in the magnetic field. Thus, simplified torso models may have a wider range of validity and usefulness than was previously thought.

INTRODUCTION

In the last 15 years or so, advances in the technology of measurement of small magnetic fields along with rapid developments in signal recording and processing have been successfully combined in tackling the problems of producing clean, reliable magnetocardiograms (MCGs). As a result, there has been considerable growth of interest both in the practical area concerning the clinical diagnostic potential of the MCG and in the theoretical relationships between the cardiac sources and the magnetic fields produced outside the torso. Ref. 1 is a recent review of much of this work. Naturally, many problems face the development of this new field of interest, not the least of which concerns the question of what new information about the heart is provided by the MCG not already contained in the standard ECG (2, 3). The answer to this question relies heavily on theoretical models of cardiac generators, torso shape, and torso inhomogeneities, together with the ability to produce both forward and inverse solutions of the magnetic problem (4–7). Closely connected to this is the important question of the sensitivity of the solution of a particular model to the details of the generators and to the nature and location of surfaces of discontinuity of electrical conductivity. The same problems arise also in electrocardiography, where several investigations (8–12) have been carried out.

It is important to know the sensitivity of model calculations because of the complexity of the system, which has forced the use of highly simplified models. These may lead to erroneous conclusions if indeed the details should turn out to be important. On the other hand, if the idealized solutions are stable to small perturbations, then insights gained will have greater general validity, and approximations to the solution of the complex physical system can then proceed with much more confidence.

Some recent theoretical work (13) on the MCG, in part addressed explicitly to this stability question, did in fact reach the discouraging conclusion that the externally produced magnetic field is very dependent upon the shape of the boundary surface enclosing the cardiac sources. It is the purpose of this communication to point out numerical errors in the calculations of ref. 13, sufficient to invalidate that conclusion, so that the way is now clear for further progress in model formulations.

THE MODEL

The model studies of ref. 13 investigate the magnetic field produced by a point current dipole source, a favorite cardiac generator, placed inside a conducting spherical torso. The field is calculated as a function of position and orientation of the dipole and the calculations are extended to cover the case of a spherical inclusion with different conductivity, situated eccentrically to represent the effects of the intracavitary blood mass. By a most elegant use of vector spherical harmonics, simple expressions are derived for the multipole coefficients of both the electric and magnetic potentials, V and U, from which the electric and magnetic fields themselves can be obtained, in some cases in closed form.

For example, the dipole coefficients of U, combined into a single vector dipole moment $\mathbf{m}$, may be expressed as shown in ref. 13 by

$$\mathbf{m} = \frac{1}{2}\int \mathbf{r} \times \left[\mathbf{J}_{(i)}(\mathbf{r})\,\mathrm{d}\tau - \sum_j (\sigma_i^{(j)} - \sigma_0^{(j)})\,V(\mathbf{r})\,\mathrm{d}\mathbf{S}^{(j)}\right]. \quad (1)$$

In Eq. 1 $\mathbf{J}_{(i)}(\mathbf{r})$ is the source term, the impressed current dipole density, while $\sigma_i^{(j)}$ and $\sigma_0^{(j)}$ are the electrical conductivities inside and outside the jth surface of discontinuity, of which $\mathrm{d}\mathbf{S}^{(j)}$ is the outward-pointing area element. Eq. 1 shows that it is in general necessary to know the solution for the electric potential first, before the magnetic potential can be evaluated. An important exception to this occurs when all the boundaries are spherical and centered on the origin, in which case the vector product, which appears in every multipole coefficient, ensures that the surface terms vanish identically—a powerful result. Consequently, if the source is purely radial, there will be no magnetic field at external points, a result known for a considerable time (14), and one which shows that the MCG and the ECG may be sensitive to different aspects of cardiac sources. Clearly $V(\mathbf{r})$ is *not* zero for the case of radial dipoles in a spherically symmetric torso.

This example illustrates well the point made previously. It is tempting to generalize the result for the dipole in a sphere and to deduce that the MCG will tend not to see radially oriented sources even when the geometry begins to deviate from spherical. The validity of such an extension rests upon whether small departures from sphericity produce correspondingly small changes in the magnetic field and it is this question which has been approached in ref. 13.

The simplest distortion of a sphere, and one which transforms it into a somewhat more realistic torso shape, is to deform it into a prolate spheroid, an ellipse rotated about its major axis. As it happens, the electric potential arising from a point dipole $\mathbf{J}_{(i)}(\mathbf{r}) = \mathbf{p}\delta(\mathbf{r} - \mathbf{r}_0)$ inside a homogeneous conducting spheroid, conductivity σ, is already known (15):

$$4\pi\sigma V(\mathbf{r}) = \sum_{l=1}^{\infty}\sum_{m=0}^{l} C_{lm}(\xi_1)\,[A_{lm}(\mathbf{r}_0)\cos m\phi + B_{lm}(\mathbf{r}_0)\sin m\phi]\,P_l^m(\eta), \quad (2)$$

where with no loss of generality we have taken $\phi_0 = 0$. Here (ξ, η, ϕ) are spheroidal coordinates and $\xi = \xi_1 =$ const. defines the torso boundary, whose eccentricity $\epsilon = 1/\xi_1$.

$$A_{lm}(r_0) = (p_\xi/h_\xi)\,P_l^m(\eta_0)\,(\partial/\partial\xi)\,P_l^m(\xi_0) + (p_\eta/h_\eta)\,P_l^m(\xi_0)\,(\partial/\partial\eta)\,P_l^m(\eta_0);$$

$$B_{lm}(r_0) = (p_\phi/h_\phi)\,P_l^m(\eta_0)\,P_l^m(\xi_0);$$

$$C_{lm}(\xi_1) = [(2l + 1)/d]\,(2 - \delta_{m0})\,\{(l - m)!/(l + m)!\}^2\,\{Q_l^m(\xi_1)$$

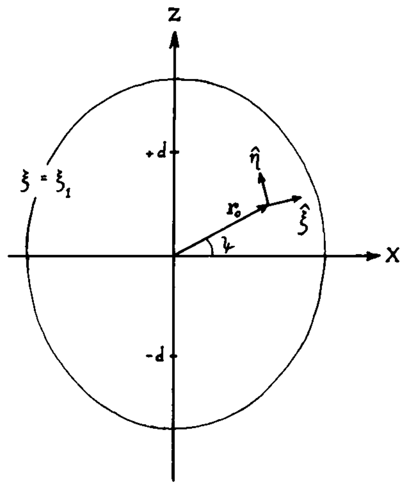

FIGURE 1 Spheroidal coordinates in the plane $\phi = 0$, showing the position of the source dipole $\mathbf{r}_0$ inside the spheroidal boundary $\xi = \xi_1 = \text{const.}$

$$- [\partial/\partial\xi \, Q_l^m(\xi_1)/\partial/\partial\xi \, P_l^m(\xi_1)] P_l^m(\xi_1)\};$$

$$h_\xi = \mathrm{d}[(\xi_0^2 - \eta_0^2)/(\xi_0^2 - 1)]^{1/2};$$

$$h_\eta = \mathrm{d}([\xi_0^2 - \eta_0^2]/[1 - \eta_0^2])^{1/2};$$

and

$$h_\phi = \mathrm{d}\,(\xi_0^2 - 1)^{1/2}(1 - \eta_0^2)^{1/2}.$$

The interfocal distance is $2d$. In the limit in which prolate spheroidal coordinates become ordinary spherical coordinates, it may be shown that Eq. 2 reduces to the known solution for a dipole in a sphere given by Frank (16). A radial dipole $\mathbf{p}$ pointing in the direction of $\mathbf{r}_0$ (see Fig. 1) has no ϕ component so that the B_{lm} are not involved in this case.

A convenient comparison with the spherical case may be made by evaluating the moment $\mathbf{m}$ from Eq. 1, in which the source term is still identically zero. Thus from Eq. 2, if ψ is the angle between $\mathbf{r}_0$ and the x axis, it is found that

$$\mathbf{m} = - \hat{\boldsymbol{\phi}}(\mathbf{r}_0)\,(pr_0/2)\,[\sin 2\psi/(2/\epsilon^2 - 1)], \tag{3}$$

which of course arises purely from the surface term in Eq. 1. The corresponding result in ref. 13 is their Eq. 51, which gives $\mathbf{m}$ larger than Eq. 3 by a factor of 12. The discrepancy in the two results is apparently due partly to the omission in ref. 13 of the factor $4\pi\sigma$ on the left side of Eq. 2, and partly to other algebraic errors in carrying out the surface integral.

DISCUSSION OF RESULTS

Eq. 3 shows that $\mathbf{m}$ does indeed become zero as $\epsilon \rightarrow 0$, the spherical case, and to see whether a small departure from sphericity produces a small change in $\mathbf{m}$, Eq. 3 may be normalized to the maximum value m could have in a sphere for any orientation of $\mathbf{p}$, namely when $\mathbf{p}$ is transverse. Then $m_{\max} = pr_0/2$ and Eq. 3 becomes

$$m/m_{\max} = \sin 2\psi/(2/\epsilon^2 - 1) \equiv D \sin 2\psi \tag{4}$$

The amplitude D of this periodic function of the angular position of the radial source dipole is independent both of p and r_0 and may be expressed in terms of the ratio $\nu = R/a$ of the radius R of a sphere to the semimajor axis a of a spheroid of equal volume.

$$D = (1 - \nu^3)/(1 + \nu^3), \quad 0 \leq \nu \leq 1, \tag{5}$$

which has a slope of $-3/2$ at $\nu = 1$, the sphere case. It has further been demonstrated that for the two independent transverse orientations of $\mathbf{p}$, the relative *change* in m, which still comes solely from the surface term in Eq. 1, is again periodic in ψ with amplitude given by Eq. 5.

From this it is clear that a small (say 1%) change in geometry, characterized by the variable ν, produces a comparably small (1-1/2%) change in m, demonstrating that the magnetic field solution is indeed stable to perturbations of the geometry. This is in sharp distinction to the conclusion of ref. 13, where a change in ν was found to produce 20 times the effect on D.

The use of Eq. 5 is not restricted to small deviations from sphericity and in fact shows for a value of ν which approximates the human torso ($\nu \sim 0.7$), that $D \sim 0.5$. The important conclusion to be drawn is that radial and transverse dipoles in a realistic geometry should make comparable contributions to the MCG, and further that these are *not* strongly affected by small changes in torso geometry, such as are produced for example by respiration.

I would like to acknowledge the continuing encouragement of Professor D. Farrell, who introduced me to the area of magnetocardiography.

This work was supported by National Institutes of Health Grant RR-07113-09, and also by a grant-in-aid from the American Heart Association, North-East Ohio Affiliate.

Received for publication 28 December 1976.

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